MATHCAMP6 PROGRAM

1. Conference, Djamel

- **5- hour introduction to the essential of quantum mechanics.**
  This series of five short lectures introduces the main postulates of quantum mechanics and some of their consequences at a very elementary level. Particular emphasis is placed on the concept of quantum entanglement, its consequences, and its applications.

2. Mini-courses

- **An introduction to abstract nonsense, (Amira + Moha)**
  You may have noticed that when studying any mathematical theory, the first thing you do is define the objects you're interested in, and the maps between them. You may then be interested in notions such as injectivity, isomorphisms, products etc.. This suggests that all theories share the same spirit, and can be expressed in terms of a universal language. This language is what we call category theory, or abstract nonsense for the experts. Many modern theories in geometry, topology and algebra are written in this language, which is why learning it early on will make your life substantially easier. In this minicourse we will explore the basics of category theory, with a focus on abelian categories.
- **Random walks on groups, (Lamine + Souheib)**
  Random walks are among the most fundamental probabilistic processes and provide a powerful tool for studying the structure of groups. Classical results show that a simple random walk on $\mathbf{Z}$ or $\mathbf{Z}^2$ almost surely returns to its starting point, while in $\mathbf{Z}^n$ for $n \geq 3$. the walk is transient and escapes to infinity with positive probability. This contrast already illustrates how probabilistic behavior can reflect the underlying geometry of the space.

  In this mini-course, we study random walks in the more general setting of finitely generated groups. We begin with an introduction to the geometric viewpoint on groups via Cayley graphs and metric properties. After reviewing the necessary probabilistic notions for random walks, we explore key examples and fundamental properties of random walks on groups.

  Particular emphasis will be placed on two major dichotomies for groups reflected in the behavior of random walks: recurrence versus transience, and the speed of escape (or drift) of the walk.

3. Homeworks

- **Homework1, (Azzedine)**
  On random walks, the homework can be found here.

- **Homework2, (Moussa)**
  About the prime number theorem, the homework can be found here.

4. Talks

- **Intrinsic Geometry of Surfaces and the Gauss–Bonnet Theorem, (Sami+Abdennacir)**
  In this talk we will explore the relationship between geometry and topology through the study of surfaces. We begin by introducing several basic notions from differential geometry. In particular, we recall the definition of regular surfaces in $\mathbb{R}^3$, which will serve as the main objects of our study. We then introduce the notion of tangent planes, which will allow us to understand directions and velocities of curves on a surface, and we discuss the fundamental forms, which provide a way to measure lengths, angles, and curvature on surfaces.
  With these tools in place, we present the remarkable result known as the Theorema Egregium of Carl Friedrich Gauss, which shows that Gaussian curvature is an intrinsic property of a surface and depends only on its first fundamental form.
  We then turn to the geometry of geodesic triangles, where the sum of the angles reflects the curvature of the surface. These observations lead to the statement and interpretation of the Gauss–Bonnet Theorem, which establishes a deep connection between the integral of curvature over a surface and the Euler characteristic, a topological invariant. Finally, we briefly discuss how this result relates to the Classification Theorem for Topological Surfaces.
- **A tour of the mathematical surfaces exhibition, (Hamza)**
- **TBD, (Ghani)**
- **The Baire Theorem and its Applications, (Anes + Rayan)**
  The Baire Theorem is a fundamental tool in analysis and topology, with important applications in functional analysis. In the first part of the talk we introduce the necessary definitions: nowhere dense sets, first and second category, and we present the statement and proof of Baire's theorem. In the second part we give some of its applications including Uniform boundnesse theorem, Open mapping theorem, and the existence of continuous nowhere differentiable functions.
- **Review of the Fourier transform and the Pontryagin duality, (Oussama)**
  On finite dimensional complex vector spaces, a set of matrices admit a simultaneous diagonalization by an orthonormal basis if and only if the matrices are normal (i.e. $AA^* = A^*A$) and they commute with each other. This is, in particular, true if the matrices are unitary or Hermitian. But, is this still true in infinite dimensions? In this talk, we consider an important example: the Hilbert space of L^2 functions, on the cirlce $\mathbb{S}^1$ or the real numbers $\mathbb{R}$, together with a natural set of commuting unitary operators: the translation operators. Can we find an orthonormal basis composed of common eigenvectors of these operators? The answer is, in some vague sense, yes! And it is given by the Fourier series/transform!

Reviewing the Fourier transform from this point of view allows us to consider its generalization to any locally compact abelian group. This is the content of the Pontryagin duality.