

MATHCAMP7 PROGRAM

1. MINI-COURSES

- **Introduction to statistics, (Houssam Boukhecham)**

Recently, interest in statistics has increased due to its practical applications in various fields, such as machine learning and artificial intelligence. Far from being a simple tool for data analysis, it is a foundational framework for solving complex problems. In this mini-course, we give a brief introduction to statistics. We begin by recalling some basic notions of probability and statistics. We then present how to simulate "random" numbers from a given distribution. After that, point estimation theory and hypothesis testing are introduced. Finally, we discuss how statistical notions can be generalized to Riemannian manifolds.

[Resources for the course.](#)

- **A Glimpse into Partial Differential Equations: Well-Posedness and Beyond, (Haroune Houamed)**

Partial differential equations (PDEs) are among the most important mathematical models in science and engineering, describing phenomena such as heat diffusion, wave propagation, fluid motion, and population dynamics. Yet, writing down a PDE is only the beginning of the story. A fundamental question is whether the equation actually provides a meaningful mathematical model: does a solution exist? Is it unique? Does it depend continuously on the data? These questions are at the heart of the notion of well-posedness, introduced by Jacques Hadamard.

This mini-course will provide an elementary introduction to the concepts of well-posedness and ill-posedness, starting from ordinary differential equations and progressively moving toward partial differential equations. We will discuss the Cauchy–Lipschitz theorem, introduce the method of characteristics through the transport equation, and illustrate several mechanisms leading to ill-posedness, including nonuniqueness, instability, and the influence of the chosen functional setting. The course will conclude with a brief discussion of the long-time behavior of solutions, highlighting the contrasting dynamics of transport and diffusion. The course is intended for undergraduate and beginning master's students and requires only a background in multivariable calculus, linear algebra, and elementary ordinary differential equations. Whenever additional mathematical notions are needed, they will be briefly recalled and introduced during the course, so that no prior knowledge of partial differential equations or functional analysis is assumed.

2. HOMEWORKS

- **Homework1 advanced, (Azzedine Saouli)**

Decomposition of a Random Variable, the homework can be found [here](#).

- **Homework2 advanced, (Adem Zeghib)**
Lie's and Engel's Theorems, the homework can be found [here](#).
- **Homework1 fundamental, (Sara Bentrifa)**
Group actions, the homework can be found [here](#).
- **Homework2 fundamental, (Billel Guelmame)**
Kolmogorov and Bernstein Inequalities, the homework can be found [here](#).

3. TALKS

- **Minkowski's Convex Body Theorem and Applications to Arithmetic, (Wassim Benlahlou)**
The Geometry of Numbers studies the interaction between continuous geometry and discrete arithmetic through the theory of lattices. One of its fundamental results is Minkowski's Convex Body Theorem, which asserts that every sufficiently large centrally symmetric convex body contains a nonzero lattice point. In this talk, we introduce the notions of lattices, convex bodies, and the determinant of a lattice, before presenting a proof of Minkowski's theorem. We then discuss classical arithmetic applications, including a theorem on small solutions of linear congruences and Fermat's Two-Square Theorem. These applications illustrate how geometric methods can be used to solve purely arithmetic problems by encoding number-theoretic information into lattices.
- **Infinite-Dimensional Optimization: An Introduction to the Calculus of Variations, (Yacine Azzeddine)**
Finding the minimum of a function is one of the oldest problems in mathematics: take a derivative, set it to zero, and centuries of theory take care of the rest. But what if the thing you're looking for isn't a number, but an entire function? What's the shortest path between two points? Why do soap bubbles have that form? Which temperature distribution does a heated object settle into? Each of these asks us to pick the "best" candidate out of an infinite-dimensional space of functions and remarkably, the familiar playbook of optimization still applies. In this talk, we'll see how ideas you already know from calculus: derivatives, critical points, descending toward a minimum all carry over to this infinite-dimensional world, and how this perspective gives us a surprisingly powerful way to discover and study the differential equations that govern physics.
- **Domaines hessiens, (Ahmed Zegloui)**
Cet exposé a pour objectif l'introduction de la notion du domaine hessien et sa relation avec la géométrie de l'information. Un domaine hessien (Ω, g) (de dimension n , $n \in \mathbb{N}^*$) est un domaine Ω de $\mathbb{R}^n$ muni d'une métrique hessienne g (i.e. une métrique riemannienne dont la matrice relativement au système de coordonnées affines usuel de $\mathbb{R}^n$ n'est autre que la matrice hessienne d'une fonction de classe C^2 sur Ω). On

parle notamment de courbure hessienne d'un domaine hessien, ainsi que le domaine hessien dual d'un domaine hessien donné. On termine par la relation entre les domaines hessiens et la géométrie de l'information, en particulier des domaines hessiens induites par des distributions normales.

- **On Quaternions, (Hichem Mammeri)**

A natural question is whether there exists a three-dimensional analogue of the complex numbers. It turns out that such an algebra cannot exist. However, the mathematician William Hamilton discovered that a four-dimensional (noncommutative) analogue does exist, namely the algebra of quaternions given by

$$\mathbb{H} = \{ a + bi + cj + dk \mid a, b, c, d \in \mathbb{R} \},$$

where the multiplication is determined by

$$i^2 = j^2 = k^2 = ijk = -1.$$

There is a standard norm on quaternions, analogous to the norm on complex numbers,

$$N(z) = z\bar{z},$$

which is defined in a similar way. One of the interesting features of quaternions is that the group of unit elements is precisely the 3-sphere $\mathbb{S}^3$, making it into a group. In fact, besides $\mathbb{S}^0$ and $\mathbb{S}^1$, which can be viewed respectively as the groups of units of the real and complex numbers, $\mathbb{S}^3$ is the only sphere that admits a topological group structure.

In this talk, we will explore the consequences of this noncommutativity on the topology and geometry of $\mathbb{S}^3$. If time permits, we will also explain how quaternions can be useful in certain physical contexts.

- **A brief discussion over the 4th and 5th colors theorems, (Benhadria Ilies Djawed)**

The four colors theorem have been a headache for mathematicians to prove for over more than a century, initially enunciated by a British mathematician and botanist Francis Gauthrie around the middle of the 19th century as a conjecture, it remained unproven until around the late 20th century and also was one of the very first theorems proven through a computer assisted proof.

First of all we will give a broad and general perspective around the implications of a such theorem, then we will see it's more rigorous formulation after presenting the necessary prerequisites in graph theory, afterwards a weaker form of this theorem will be presented being the fifth colors theorem along with it's surprisingly simple proof, finally, more details around the method involved in the four colors theorem proof as well as the theoretical implications used will be presented.

- **Special Relativity: Geometry, Time, and Minkowski Spacetime, (Walid Boutaghou)**

We all grow up believing that space and time are absolute, fixed backgrounds against which events unfold. Yet when we take the simple postulate that the speed of light is the same for all observers and follow it

to its logical conclusion, our familiar notions collapse: lengths contract, clocks slow down, and simultaneity becomes relative.

In this talk, we explore special relativity as a geometric theory built on Minkowski spacetime. Following the classical treatment of Barrett O'Neill and others, we view Minkowski spacetime as a four-dimensional manifold equipped with an indefinite metric. We show how familiar ideas from calculus and linear algebra — inner products, orthogonality, and geodesics — naturally extend to this setting. From this perspective, the Lorentz transformations arise geometrically, the twin paradox is resolved through worldlines, and Einstein's famous $E = mc^2$ emerges as a fundamental structural consequence of Minkowski geometry.

Whether you are discovering relativity for the first time or revisiting it with a mathematical lens, this talk offers a unified and elegant bridge between classical mechanics and the richer realm of general relativity.

- **Hyperbolic Crochet: Topology at our fingertips, (Karabadji Said Abdellah)**

The 19th century witnessed a profound mathematical revolution with the discovery of non-Euclidean geometries, concluding a nearly 2,000-year-old endeavor to prove Euclid's parallel postulate. Among these newly discovered spaces, hyperbolic geometry proved to be the most elusive and difficult to intuitively visualize. In this presentation, we explore how different geometries impose strict limits on how one can tessellate their respective model spaces. We will then demonstrate how specific, regular tessellations of the hyperbolic plane can be translated directly into a tactile crocheting algorithm, yielding a physical, three-dimensional model of a hyperbolic ball.